Improvised Absorbing Boundary Conditions for Two-dimensional Electromagnetic Finite Elements

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This paper briefly explains and aims to extend D. Meeker's Improvised Absorbing Boundary Conditions (IABC) for the two-dimensional high frequency electromagnetic finite elements. To further emphasize, the IABC method approximates unbounded space as series of isotropic cylindrical shells so as to emulate open boundaries without the need for additional code to any finite element solver. Previous works of these techniques had been addressed to both magneto static as well as electrostatic problems. This present work however, attempts to extend and modify the method to a two-dimensional high-frequency electromagnetic propagation as a solution for related scalar potential formulations.

***Index Terms*— Finite element methods, open boundary problem, Helmholtz equation, non-linear optimization**

I. INTRODUCTION

MOST of the high-frequency electromagnetic problems are confined to an open boundary problem. To start with, asymptotic boundary conditions are an established method of addressing "open boundary" problems that are mostly related to low-frequency problems. This method applies boundary conditions that emulate the impedance of an unbounded space for low-order harmonics at a nearby computational boundary. In line with this, a variety of techniques had been developed [1-7] to emulate open boundary problems in finite element analysis. Those techniques however, are classified into two types. In the low-frequency applications, the technique tries to emulate the infinite region**;** therefore, called "asymptotic boundary conditions" as described above. In comparison with the high-frequency applications, the scheme is to absorb the radiated energy, and can therefore be called the "absorbing boundary conditions". To shed more light upon the matter, the "absorbing boundary conditions" concentrate on the properties of the electromagnetic waves and that therefore, only applicable to high-frequency problems**.** Upon comparison then, we will see that the "asymptotic boundary conditions" are based on the properties of Laplace equations, but not on Helmholtz equations, and is therefore, only applicable to low-frequency problems.

Recently, improvised asymptotic boundary conditions (IABCs) were proposed [6, 7]. This method approximated unbounded space as series of isotropic cylindrical shells so as to emulate open boundaries. Those cylindrical shells, which can be considered as electrical images, work on the same principle with that of the asymptotic boundary conditions, in the same way that it reproduces the impedance of an unbounded region for low-order harmonics. However, the negligible error became eminent when the modeling of the boundary impedance for higher order harmonics became inexact and inaccurate. To further state this, the first-order ABCs specify the parameters of a mixed-type boundary condition that matches the far-field characteristics of a magnetic dipole. In this case, higher order ABCs that emulate the far-field behavior of multi-poles are possible and provide improved accuracy. The biggest advantage of this method is that it requires no additional code to any finite element solver.

In this paper, we attempt to extend the method to two-dimensional high-frequency electromagnetic wave propagation problems. A Third-order ABC in two-dimensional wave propagation with finite element problems was presented. We have employed the finite element solver for general purpose FreeFEM++@ and have demonstrated that the accurate open boundary solutions can be obtained.

I. IMPROVISED BOUNDARY CONDITION

A. *Improvised Boundary Condition Formulation*

Figure 1 shows a schematic image of the third-order improvised absorbing condition. The region of interest is the inner boundary and the outer shells work as absorbers. The exterior boundary is a PEC (Perfect Electric Conductor).

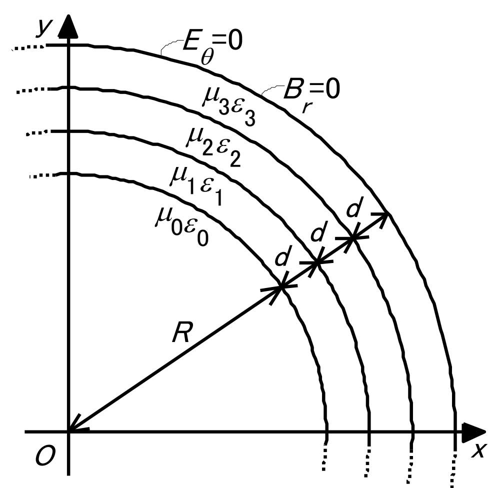

Fig. 1. Schematic image of the third-order three-dimensional improvised absorbing condition. The region of interest is the inner boundary and the outer shell works as an absorber. The exterior boundary is a PEC.

The continuity conditions (1) must hold at each boundary of the adjacent two shells. At the exterior boundary, the boundary condition is given as (2)

$$
\begin{cases} D_r^{n+1} = D_r^n \\ E_\theta^{n+1} = E_\theta^n \\ E_\phi^{n+1} = E_\phi^n \end{cases} \quad \begin{cases} B_r^{n+1} = B_r^n \\ H_\theta^{n+1} = H_\theta^n \\ H_\phi^{n+1} = H_\phi^n \end{cases} \tag{1}
$$

$$
\begin{cases} E_\theta = 0 \\ B_r = 0 \end{cases} \tag{2}
$$

Electromagnetic fields in vacuum can be expanded with the Henkel functions. We can enforce the solution inside the interior cylinder to be second kind of the Henkel function which is the out-going wave, by choosing the appropriate permeability and permittivity in each shell. With the three-layer IABC, the third-order approximate open boundary solution (dipole, quadrupole and sextupole) can be obtained.

II. Numerical Examples

We employed an optimization procedure to find the solution of the continuous equations (3) with the boundary conditions (2). We obtained permeability and permittivity of each shell, given in (4), with the normalized frequency of $\omega/c = 6.0$.

$$
\begin{cases} \mu_{m+1}\psi_{m+1}^{\mathrm{TM}}(k_{m+1}r_m) = \mu_m\psi_m^{\mathrm{TM}}(k_m r_m) \\ \dfrac{d}{dr}\psi_{m+1}^{\mathrm{TM}}(k_{m+1}r_m) = \dfrac{d}{dr}\psi_m^{\mathrm{TM}}(k_m r_m) \\ \varepsilon_{m+1}\psi_{m+1}^{\mathrm{TE}}(k_{m+1}r_m) = \varepsilon_m\psi_m^{\mathrm{TE}}(k_m r_m) \\ \dfrac{d}{dr}\psi_{m+1}^{\mathrm{TE}}(k_{m+1}r_m) = \dfrac{d}{dr}\psi_m^{\mathrm{TE}}(k_m r_m) \end{cases} \tag{3}
$$

$$
\begin{cases} \mu_1' - j\mu_1'' = 0.345038 + 0.008230\,j \\ \mu_2' - j\mu_2'' = -0.362825 + 0.166794\,j \\ \mu_3' - j\mu_3'' = 0.471253 + 1.185403\,j \\ \varepsilon_1' - j\varepsilon_1'' = 1.049472 - 0.465843\,j \\ \varepsilon_2' - j\varepsilon_2'' = 1.011375 - 4.278877\,j \\ \varepsilon_3' - j\varepsilon_3'' = -2.539863 - 0.487945\,j \end{cases} \tag{4}
$$

To verify the method, we employed a small dipole model and analyzed with the FEM solver FreeFEM++@. Figure 3 and 4 shows the comparison of ψ^{TM}, and ψ^{TE} obtained by analytic formula and the FEM. They show the exact match in the region inside the interior cylinder, which means the IABC is also valid to high frequency problems.

III. Conclusion

The faster the scalar potentials decay with the ascending distance, then the higher the order will be. Therefore, if the radius of the region of interest is large enough, the reflection of the higher order waves will be negligible. Furthermore, any electromagnetic field distributions can be expressed with the liner combination of multi-poles, thus an approximate open boundary can be obtained with the conventional finite element solver, with which no special technique for the open boundary problems were implemented.

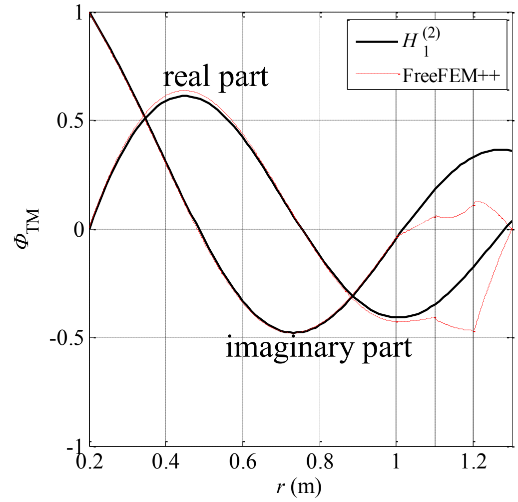

Fig. 2.Comparison of the scalar potential ψ^{TM} (dipole, n=1) obtained by the Hankel function of the first kind, and the field distribution obtained by FreeFEM++ along the Y-axis. Exact match was observed in the region r < 1.0. The exterior boundary is the Dirichlet boundary condition.

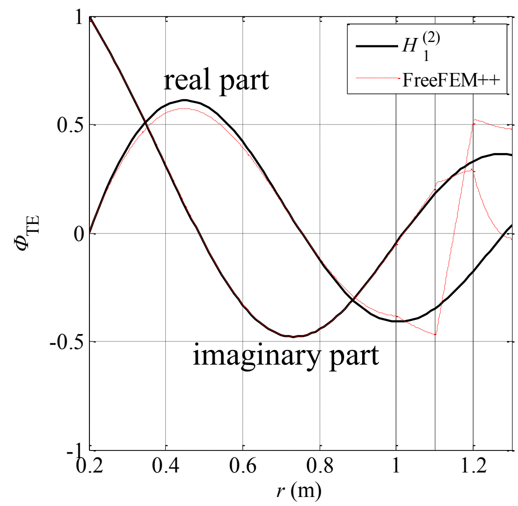

Fig. 3. Comparison of the scalar potential ψ^{TE} (dipole, n=1) obtained by the Hankel function of the first kind, and the field distribution obtained by FreeFEM++ along the Y-axis. The exact match was observed in the region r < 1.0. The exterior boundary is the Neumann boundary condition.

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